Lecture 20

Wednesday, October 26, 2016 9:12 AM

4.9 Antiderivatives

A func F is called an antiderivative of func f on an interval I if $F'(x) = f(x)$ for all x in I.

Ex $f(x) = 3x^2$. Find it's antiderivative.

$F(x) = x^3$, $F'(x) = 3x^2 = f(x)$

So F is an antiderivative of $f(x)$

$G(x) = x^3 + 1$, $G'(x) = 3x^2 = f(x)$

So G is also an antiderivative of $f(x)$.

In general, any function $H(x) = x^3 + C$ (C is a const) is an antiderivative of $f(x) = 3x^2$.

Are there any other ?

- From MVT class, we showed that if two functions have the same derivative on an interval, then they must differ by a constant.

- If F and G are antiderivatives of f, $F'(x) = f(x) = G'(x)$

  $\Rightarrow G(x) = F(x) + C$

<u>Theorem</u> If F is an antiderivative of f on an interval, then the most general antiderivative of f on this interval is $F(x) + C$, C is an arbitrary constant.

<u>NOTATION</u> $\int f(x)\, dx$ is the notation for antiderivative of $f(x)$.

i.e $\int f(x)\, dx = F(x)$ means

$$F'(x) = f(x)$$

<u>TABLE OF ANTIDERIVATES</u>

C = arbitrary constant

$$\int K\, dx = Kx + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad \underline{(n \neq -1)}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{1}{n+1} \frac{d}{dx}\left(x^{n+1}\right)$$

$$= \frac{1}{\cancel{n+1}} \cdot \cancel{(n+1)} \cdot x^{n+1-1}$$

$$= x^n$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int b^x \, dx = \frac{b^x}{\ln b} + C \quad (b > 0)$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int c f(x) \, dx = c \int f(x) \, dx$$

$$\int f(x) \pm g(x) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Ex Find the general antiderivative of

$$g(x) = 6x^5 + \frac{6}{\sqrt{1-x^2}} + 14^x$$

$$\int g(x)\,dx = \int 6x^5 + \frac{6}{\sqrt{1-x^2}} + 14^x\,dx$$

$$= 6\int x^5\,dx + 6\int \frac{1}{\sqrt{1-x^2}}\,dx + \int 14^x\,dx$$

$$= 6 \cdot \frac{x^{5+1}}{5+1} + 6\sin^{-1}x + \frac{14^x}{\ln 14} + C$$

$$= x^6 + 6\sin^{-1}x + \frac{14^x}{\ln 14} + C$$

Ex If $f'(x) = \dfrac{2\sqrt{x^3} + \sqrt[4]{x^5}}{\sqrt[6]{x}}$ and $f(1) = \dfrac{59}{175}$

Find $f(x)$.

Soln $f'(x) = \dfrac{2x^{3/2} + x^{5/4}}{x^{1/6}}$

$$= \frac{2x^{3/2}}{x^{1/6}} + \frac{x^{5/4}}{x^{1/6}}$$

$$= 2x^{3/2 - 1/6} + x^{5/4 - 1/6}$$

$$f'(x) = 2x^{4/3} + x^{13/12}$$

$$\int f'(x)\,dx = \int 2x^{4/3} + x^{13/12}\,dx$$

$$= 2\int x^{4/3}\,dx + \int x^{13/12}\,dx$$

$$= 2 . \frac{x^{4/3 + 1}}{4/3 + 1} + \frac{x^{13/12 + 1}}{13/12 + 1}$$

$$= \frac{2x^{7/3}}{7/3} + \frac{x^{25/12}}{25/12} + C$$

$$= \frac{6}{7}x^{7/3} + \frac{12}{25}x^{25/12} + C$$

$$f(x) = \frac{6}{7}x^{7/3} + \frac{12}{25}x^{25/12} + C$$

To determine C we use the fact that

$$f(1) = \frac{59}{175}$$

$$f(1) = \frac{6}{7} + \frac{12}{25} + C$$

$$\frac{59}{175} = \frac{150 + 84}{175} + C$$

$$\frac{59}{175} - \frac{234}{175} = C$$

$$-\frac{175}{175} = C \Rightarrow C = -1$$

$$f(x) = \frac{6}{7}x^{7/3} + \frac{12}{25}x^{25/12} - 1$$

Ex $f''(x) = \cos x + \sin x$,

$f'(0) = 2$, $f(0) = 4$. Find $f(x)$.

$$\int f''(x)\,dx = \int \cos x + \sin x \, dx$$

$$f'(x) = \sin x - \cos x + C$$

- $f'(0) = \sin 0 - \cos 0 + C$

$$2 = 0 - 1 + C \Rightarrow C = 3$$

$$f'(x) = \sin x - \cos x + 3$$

$$f(x) = \int f'(x)\,dx$$

$$= \int \sin x - \cos x + 3 \, dx$$

$$f(x) = -\cos x - \sin x + 3x + D$$

$$f(0) = -\cos 0 - \sin 0 + 3.0 + D$$

$$4 = -1 + D \Rightarrow D = 5$$

$$f(x) = -\cos x - \sin x + 3x + 5$$

Notation $s(t) \equiv$ position
$v(t) \equiv$ velocity
$a(t) \equiv$ acceleration.

$$s'(t) = v(t)$$
$$v'(t) = a(t) \Rightarrow s''(t) = a(t)$$

Ex A particle moves in a straight line and has acceleration $a(t) = 6t + 4$. It's initial velocity, $v(0) = -6$ cm/s and it's initial position, $s(0) = 9$cm. Find $s(t)$.

Soln $a(t) = v'(t) = 6t + 4$

$$v(t) = \int a(t)\,dt = \int 6t + 4\,dt$$
$$= \frac{6t^2}{2} + 4t + C = 3t^2 + 4t + C$$
$$v(0) = 3.0^2 + 4.0 + C$$
$$\Rightarrow -6 = C$$
$$v(t) = 3t^2 + 4t - 6$$

$$s'(t) = v(t) = 3t^2 + 4t - 6$$

$$s(t) = \int 3t^2 + 4t - 6 \; dt$$

$$= t^3 + 2t^2 - 6t + D$$

$$s(0) = 9 \Rightarrow D = 9$$

$$s(t) = t^3 + 2t^2 - 6t + 9$$